The Steady-state Analysis of DC Distribution Network Embedded Droop Control and Power Flow Controller

Da Xie, Aikang Chen, Songtao Yu, Chenghong Gu, Yan Li

***Abstract*—Aiming at the DC distribution network based on the droop control, this paper proposes that the intersection of the internal characteristics of the node converter droop control and the external characteristics derived from the parameters of the DC distribution network is the actual node operation point. The quantitative relationship between the droop control characteristic parameters and the converter functions is discussed, and the comprehensive droop control characteristics of the multi-converter node is analyzed. The automatic adjustment method of droop control, containing the secondary adjustment and the third adjustment, is presented, and the power flow algorithm of DC distribution network, which considers the impact of power flow controller, droop control and automatic adjustment method, is derived. Finally, three cases about power flow controller, the secondary adjustment and the third adjustment are used to verify that droop control and its related regulation methods can optimize the operation conditions of DC distribution network.**

***Index Terms*—Flexible DC distribution network, droop control, automatic adjustment method, steady-state analysis**

1. INTRODUCTION

THE DC transmission technology has become a significant supplement of the AC transmission system since the first DC transmission project was put into operation in Gotland Sweden, 1954 [1,2]. In the modern power distribution system, the development of distributed sources, energy storage devices and loads promote the development of flexible DC distribution network. Photovoltaic batteries, wind power generators, fuel cells and gas turbines are most common distributed sources, whose final electric outputs are DC or can be simply rectified into DC. If these distributed sources are connected to the DC distribution network, DC-AC conversion links can be avoided, which means lower costs, reduction of power losses and higher power conversion efficiency [3]. On the other hand, there is an increasing number of DC loads in the modern distribution network, such as electric vehicles and electronic devices. Besides, AC-DC-AC conversions are commonly used for some AC loads like pumps and air-conditioners to improve power quality [4, 5]. DC-AC links can also be saved if these loads are connected to the DC distribution network.

Currently, research on the control methods of the flexible DC distribution network mainly focuses on converters or micro-grid control methods, but there are no mature control strategies for DC distribution network. Referring to the control methods of DC transmission, the main feasible methods of flexible DC distribution network are master-slave control, droop control and voltage margin control [6-8]. Compared to master-slave control and margin control, the droop control method can realize flexible grid connection, off-grid and sub-network operation, and it has strong anti-interference ability to system topological changes [9-11], which is suitable for the characteristics of the flexible DC distribution network.

The existing research on droop control is mainly focused on the optimization and coordination control methods of AC systems, and the related research and application have become relatively mature. The work in [12] proposed the droop control methods for parallel inverter operation in islanded and grid-connecting modes, which can realize the control of frequency and voltage, and mitigate voltage harmonics. Droop control framework and control methods are presented in [13-15] for the grid and islanded operation of AC micro-grid. It can reduce communication bandwidth and control costs, and realize smooth switch between several operation modes. Aiming to achieve accurate active power distribution, the work in [16-17] proposes an adaptive voltage droop control strategy, which is not affected by communication delay. The research on the droop control of DC distribution grid, which is relatively few, is mainly focused on the operation control of DC micro-grids. Research in [18] proposes a robust droop control method by combining local information and voltage control, and the work in [19,20] presents a distributed droop control method, realizing optimized power dispatching of DC micro-grid. However, these droop control strategies need large-scale adjustments and are focused on the adaptive dispatch. The objectives of them are various optimizations, e.g., the minimum loss, and the minimum operation cost. In this paper, considering the deviation of droop control, a droop control method is presented, which fine tunes the bus voltage and power to decrease the deviation and ensure that the bus voltage and power remain in the permitted range. And the proposed method is a passive adjustment in small scale.

The main contributions of this paper are shown as follows:

- This paper proposes the design range of droop slope and quantifies the relationship between the slope and the function of the converter, which provide the evidence for setting the droop slope of a practical converter.
- Based on analysis of multi-converters' parallel operations on a bus, an automatic adjustment method of droop control is developed, which could fine tune bus voltage and power to meet bus requirements of DC distribution network.
- Considering droop control of converters and automatic adjustment method, the power flow algorithm of DC distribution network is derived, which underlies further research on dispatch strategies and energy management.

The rest of this paper is as follows: Section II discusses the droop control characteristics of DC converters. Section III develops

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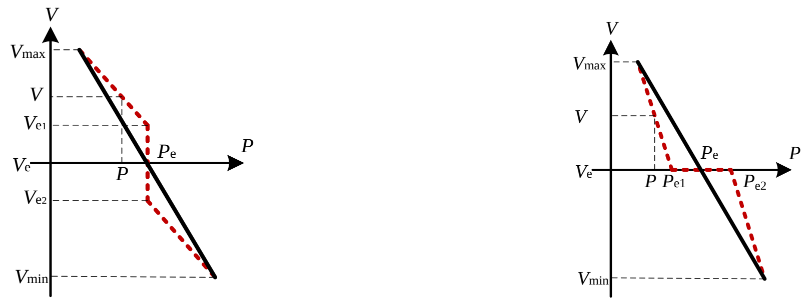
the automatic adjustment method of droop control. In section IV, considering droop control and automatic adjustment, power flow equation and its solution algorithm of the DC distribution network are studied. In section V, a testing DC distribution network is applied to demonstrate the analyses in section III and IV with three cases. The conclusions are drawn in section VI.

2. Droop control characteristics of DC converters

2.1. Basic operation mechanism

The droop control is a control method that controls the voltage drop at the converter terminal voltage with the power variation. It is a different adjustment control method. Fig.1 shows the power-voltage characteristic curve of the droop control, where the solid part is the conventional droop control and the dashed line is the droop control with a dead zone [21]. The Fig. 1(a) shows the buses adopt the droop control with voltage dead zone. The droop control in Fig. 1(b) is suitable for the bus with constant voltage control. This kind of bus acts generally as balance bus in DC network and has a large spare capacity.



(a) Voltage dead zone (b) Power dead zone

Fig. 1 Voltage-power characteristic of converter droop control

When using the droop control with a dead zone, the converter can achieve non-difference adjustment, in a certain voltage range of power adjustment, as shown in Fig.1(a) (or in a certain power range of voltage adjustment, as shown in Fig.1(b)).

In the droop control, when the slope of droop control k=0, bus voltage V=V_e, when k=+∞, output power P=P_{e1}. Obviously, constant voltage control and constant power control can be considered as two special cases of droop control.

2.2. Quantitative relationship between droop characteristics and converter functions

Regarding different demands for distributed energy and load operation characteristics, the droop control characteristic of converter connecting to DC distribution network has various parameter requirements. Thus, there exists the quantitative relationship between the converter function and the droop characteristic parameters. The following analyses give approaches to the droop control parameters designing for the slack bus, source bus and load bus.

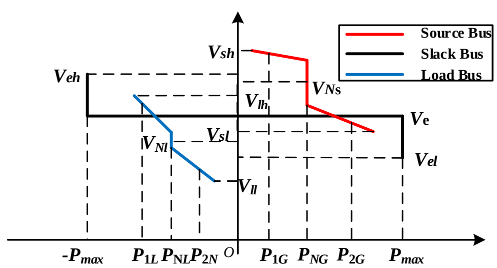


Fig. 2 Droop control characteristics of different bus types

1) Droop characteristic parameter for slack bus

The droop characteristic of the slack bus converter is shown in Fig. 2 as the black curve. V_e is the operation voltage, P_{max} is the maximum power limit, and V_{eh} and V_{e1} are the upper and lower permitted operation voltage.

Under normal operation conditions, the slack bus has to operate at fixed voltage V_e, and the droop slope of slack bus converter should be k=0 with zero voltage dead zone. When the output or input power reaches the maximum power limit, the converter starts to operate under fixed power control mode.

2) Droop characteristic parameter for source bus

The droop characteristic of the source bus converter is shown in Fig. 2 as the red curve, where P_{NG} is the rated output power of the source bus. P_{NG}>0 indicates that the source bus supplies power to the grid. P_{1G} and P_{2G} are the upper and lower limit of source power fluctuation.

As the modern distribution network generally involves some distributed renewable energy, including fluctuating energies like wind power and solar energy, the stochastic fluctuation problem of source bus has to be considered. The permitted operation voltage range is defined as ΔV_N, the dead zone range is ΔV_d, and the power fluctuation range is ΔV_G. In order to meet the requirement of the normal operation voltage range, droop slope of the source bus converter k has to satisfy:

$$-\left(\Delta V_N - \Delta V_d\right)/\Delta P_G \le k \le 0 \tag{1}$$

The maximum power adjustment range is:

$$\Delta P_{G\max} = -(\Delta V_N - \Delta V_d)/k \tag{2}$$

It is clear from (2) that the converter will have higher power adjustment range with lower droop slope.

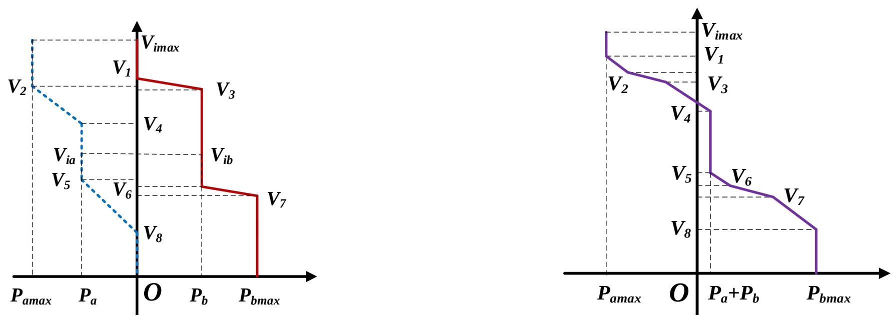
3) Droop characteristic parameter for load bus

The droop characteristic of load bus converter is shown in Fig. 2 as the blue curve, where P_{NL} is the rated output power of load bus. $P_{NL}<0$ indicates load bus absorbs power from the grid. P_{1L} and P_{2L} are the upper and lower limit of power fluctuation.

Similar to the source bus, since the user load is fluctuating on the time scale, the fluctuation of the load power should also be taken into account when designing the droop characteristic parameters of the load bus converter. Here, we do not repeat it.

2.3. Parallel operation of multi-converters

For the source or load buses in the DC distribution network, the multi-converters may operate in parallel on the same bus, and the droop control characteristics of different converters are usually not the same due to different operating characteristics of sources or loads. Here, we take the parallel operation of mixed sources and loads as an instance, and it has a variety of possible operating states.



(a) All droop control characteristics (b) Overall droop control characteristics

Fig. 3 Parallel operating of source and load

The detailed droop control characteristics are shown in Fig. 3(a). Where V_{ia} and V_{ib} are the converter DC side voltages, P_{ia} and P_{ib} are the output power, P_{amax} and P_{bmax} are the maximum power limit of the two converters, and V_{iamx} is the maximum voltage limit of bus i. the voltage of bus i has to satisfy $V_i=V_{ia}=V_{ib}$, when the two sources operate in parallel, and the input power of bus i is $P_i=P_{ia}+P_{ib}$.

Assuming that droop slope of converter a and b are k_a and k_b, expected power are P_a and P_b, the voltage-power relationship of bus i is (3).

$$P_i = \begin{cases} P_{b\max} & V_9 \le V_i < V_8 \\ P_a + (V_i - V_5)/k_a + P_{b\max} & V_8 \le V_i < V_7 \\ P_a + (V_i - V_5)/k_a + P_b + (V_i - V_6)/k_b & V_7 \le V_i < V_6 \\ P_a + (V_i - V_5)/k_a + P_b & V_6 \le V_i < V_5 \\ P_a + P_b & V_5 \le V_i < V_4 \\ P_{a\max} + (V_i - V_2)/k_a + P_b & V_4 \le V_i < V_3 \\ P_{a\max} + (V_i - V_2)/k_a + (V_i - V_1)/k_b & V_3 \le V_i < V_2 \\ P_{a\max} + (V_i - V_1)/k_b & V_2 \le V_i < V_1 \\ P_{a\max} & V_1 \le V_i < V_{i\max} \end{cases} \tag{3}$$

The overall equivalent droop control characteristic curve of bus i is shown in Fig. 3(b). Under normal circumstances, multi-converters in parallel should operate at the power setting values, which means all converters operate in the dead zones and satisfy $V_5 \le V_{ia}=V_{ib}<V_4$. However, constrained by the power-voltage characteristic of the DC distribution network, converters also operate in other zones of droop control curves.

3. Automatic adjustments of droop control

3.1. Characteristic of droop control considering grid parameter

There are only DC components in the DC distribution grid. Therefore, output or input power of each bus is determined by the bus voltage, adjacent bus voltages and connection line resistances. As is shown in Fig. 4, for the i^{th} bus of DC distribution network, V_i is the voltage of bus i, V_1, V_2, …, V_n are the adjacent bus voltages, and R_{i1}, R_{i2}, …, R_{in} denote the connection line resistances between i^{th} and the adjacent buses.

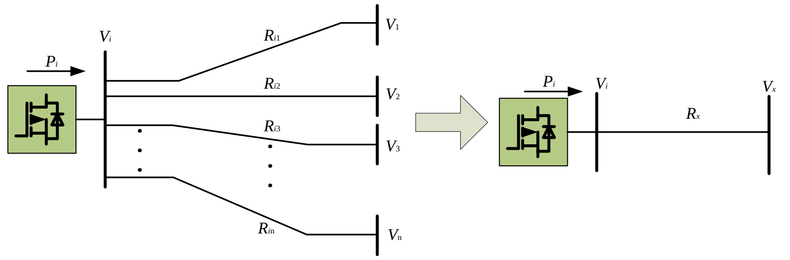


Fig. 4 Equivalent model of single-terminal flexible DC distribution network

From Fig. 4, the output power of i^{th} bus can be calculated as:

$$P_i = V_i \sum_{j=1}^{n} \left(V_i - V_j\right) / R_{ij} \tag{4}$$

The adjacent buses of bus i can be equivalent to a single bus x. The voltage of bus x is V_x and the resistance between bus i and bus x is R_x. The output power of bus i can also be expressed as:

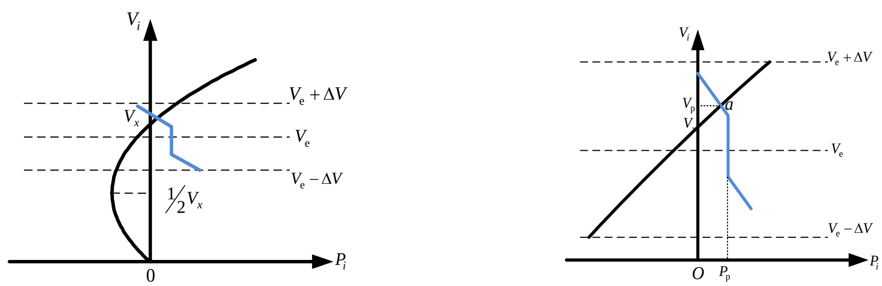
$$P_i = V_i \left(V_i - V_x\right) / R_x = \left[(V_i - 0.5V_x)^2 - 0.25{V_x}^2\right] / R_x \tag{5}$$

The relationship of V_i and P_i in (5) and the droop control characteristic are shown in Fig. 5(a), where V_e denotes the base voltage and ΔV denotes the permitted voltage fluctuation.

Determined by the droop characteristic and grid parameter, the actual operation point is depicted in Fig. 5. The two curves are respectively the internal characteristic (characteristic of droop control) and external characteristic (characteristic of network operation). To ensure the normal operation of the DC distribution network, bus voltage should be limited as shown in Fig. 5(b). Because the voltage range ΔV is far less than V_e, the external characteristic within the prescribed limits approximates to a straight line, the slope of which is the slope of the tangent line at the point (0, V_x). On the basis of (5) the equivalent slope k_2 of operation characteristic can be derived as follows:

$$k_2 = \frac{dV_i}{dP_i}\bigg|_{(0,V_x)} = \frac{R_x}{2V_i - V_x}\bigg|_{(0,V_x)} = \frac{R_x}{V_x} \tag{6}$$

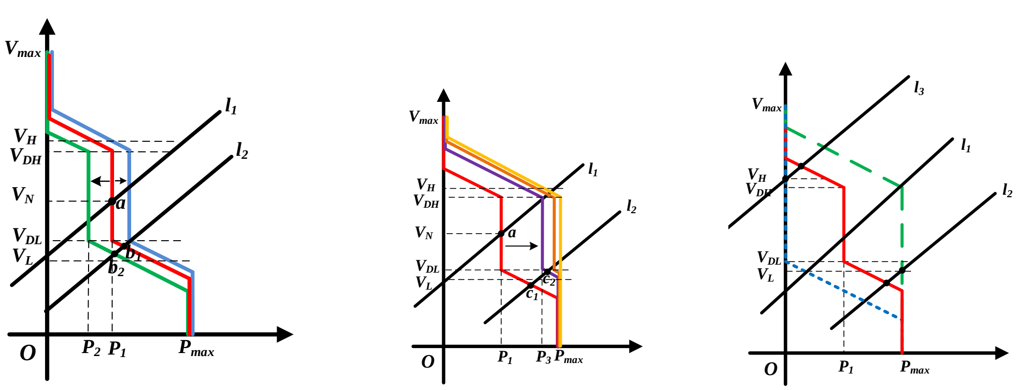
When the converter reaches the steady operation point, the intersection a of the two curves in Fig. 5 is the actual running point of bus i. V_p and P_p are the actual bus voltage and power of the converter.



(a) Internal and external characteristic (b) Permitted operating characteristic

Fig. 5 Operating characteristic of a bus in flexible DC distribution network

3.2. Automatic adjustment method of droop control



(a) Case 1 (b) Case 2 (c) Secondary adjustment range

Fig. 6 Secondary adjustment of the automatic adjustment method

As is analyzed previously, multiple converters with different characteristics of droop control may operate on the same bus, and the overall internal characteristic of the bus tends to be a droop control function with multiple segments. When this bus is connected to the DC distribution network, its actual operation point may locate in different zones. Based on the concept, this paper develops an automatic adjustment method of droop control with voltage dead zone, containing the secondary adjustment and the third adjustment, to mitigate the power and voltage deviations. For the clarity of the method, a five-segment curve of overall droop control is applied here as an example to display the automatic adjustment method of droop control with voltage dead zone, which is shown in Fig. 6 and Fig. 7. V_N is the rated power capacity of bus *i*, and V_L and V_H are the permitted lower and upper voltage limits. If the permitted voltage range is 3%, then V_L=0.97V_N, V_H=1.03V_N. V_{DL} and V_{DH} are the lower and upper limits of the dead zone. If the dead zone range is 2%, then V_{DL}=0.98V_N, V_{DH}=1.02V_N. V_{max} and P_{max} are respectively the maximum voltage limit and the power limit of bus *i*. P_1, P_2 and P_3 are the power setting values before and after the secondary adjustment.

When DC distribution network is disturbed or the operation status changes, the operation characteristics of buses will change, resulting in shifting of the steady operation points of buses. Hence, in order to mitigate the power and voltage deviations, secondary adjustment pans the voltage dead zone by regulating the power setting value and keeps the slope of droop segment constant, as shown in Fig. 6. Subsequently, several typical cases of secondary adjustment are discussed qualitatively and quantitatively. In Fig. 6(a), the black curves l_1 and l_2 are the operation characteristics of bus *i* before and after the disturbance, and the red curve represents the initial droop control, so the operation point changes from *a* to b_1. At this point, the disturbance is relatively small, the voltage is in the permitted voltage range although it has exceeded the dead zone, and the power is less than P_{max}. There is a certain power deviation of b_1 from P_1, which can be mitigated by secondary adjustment. The droop control after the left shift is shown in Fig. 6(a) as a green curve. It can be seen clearly that the operation point changes from b_1 to b_2, and the power deviation is alleviated with the increase of voltage difference. This way applies to the bus that has high power requirement. Assume the power adjustment amount is δ_{p1}, the droop slope is k_1 and the slope of the grid operation characteristic is k_2, the voltage of b_1 is V_{b1}. Then the shifted setting power value P_2 of droop control and the power adjusting range can be calculated as follows:

$$P_2 = P_1 - \left(\delta_{P1} - \frac{\delta_{P1} k_2}{k_1} \right) \tag{7}$$

$$V_{b1} - \delta_{P1} k_2 / k_1 < V_L \Rightarrow \delta_{P1} < \frac{V_{b1} - V_L}{k_2} \tag{8}$$

Similarly, if the bus *i* has high voltage requirement, we can move the droop control right, as the blue curve in Fig. 6(a). And the analysis is the same as case 2 in Fig. 6(b).

In Fig. 6(b), when the disturbance of grid is relatively large, the operation point of bus *i* changes to c_1, where the operation voltage V_i<V_L, exceeding the permitted voltage range. The voltage deviation can be mitigated by secondary adjustment, as the purple, orange and blue curves illustrate. After the adjustment, the operation point changes from c_1 to c_2, which returns to the normal voltage range. Assuming the adjustment amount of voltage deviation is δ_V, the voltage of c_1 is V_{c1}, then the shifted setting power value P_3 of droop control is:

$$\begin{cases} P_3 = P_1 + \delta_V \left(\frac{1}{k_2} - \frac{1}{k_1}\right), V_L - V_{c1} < \delta_V \le V_{DL} - V_{c1} \\ P_3 = P_1 + \frac{\delta_V}{k_2} - \frac{V_{DL} - V_{c1}}{k_1}, V_{DL} - V_{c1} < \delta_V < \left(P_{\max} - P_1 + \frac{V_{DL} - V_{c1}}{k_1} \right) k_2 \\ P_3 = P_{\max}, \delta_V = \left(P_{\max} - P_1 + \frac{V_{DL} - V_{c1}}{k_1} \right) k_2 \end{cases} \tag{9}$$

Here, the first equation corresponds to the purple curve, the corresponding curve of the second equation is orange curve, and the yellow curve represents the special situation, where the setting power is P_{max}.

It's clear that the voltage and power deviations cannot be alleviated at the same time on account of the coupling characteristic in DC network. With the secondary adjustment of droop control, the specified one can be regulated according to the requirements of buses. However, the adjustment capacity is limited by the maximum power limit of the bus and the permitted voltage range. The secondary adjustment range is shown in Fig. 6(c), where l_1 and l_2 respectively represent the lower limit and upper limit of the secondary adjustment. If the disturbance is out of the limits, the secondary adjustment will fail to further reduce the voltage and power deviations.

For some buses with high power and voltage requirements, it is possible to further adjust the slope of the droop control to reduce power and voltage deviations. The slope adjustment, which is called the third adjustment in this paper, is shown in Fig. 7. The red curve and purple curve are the droop control before and after the third adjustment. Through the third adjustment, the operation point changes from a_1 to a_2 and the corresponding operating power changes from P_1 to P_2. The power deviation is alleviated after the third adjustment. Assuming that the adjustment amount of power deviation is δ_{P2}, the droop slope and the slope of grid operation characteristic are k_1 and k_2, and adjustment amount Δk of droop slope is:

$$\Delta k = \frac{\left[k_1 (P_{N1} - P_1) - k_2 \delta_{p2} \right]}{P_{N1} - P_1 - \delta_{p2}} \tag{10}$$

Similarly, through the third adjustment, the operation point changes from a_1 to a_3, and the voltage deviation is alleviated after the third adjustment.

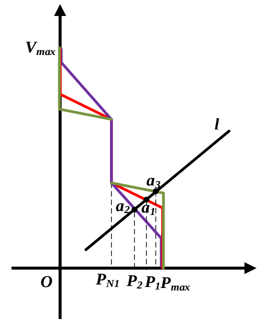


Fig. 7 The third adjustment of the automatic adjustment method

when the bus containing several droop control converters needs secondary or third adjustment, we can adjust the droop control in turn on the basis of the capacity and adjust ranger of each converter to avoid the coupling among converters.

It should be noted that the operation characteristic shall be basically unchanged after automatic adjustment method. Obviously, the variation of equivalent voltage V_x is very small when we employ the method, so the slope k_2 and the point (0, V_x) is almost invariant, which means that the black curves in Fig. 6 and Fig. 7 have a minor shift. Hence, we can compare the internal and external curves in each iteration of power flow to determine whether or not execute automatic adjustment method and the specified adjustment values. Finally, after enough iterations, the power flow of the DC distribution network will reach requirements of all buses, as shown in Fig. 8. And this issue will be discussed in more detail in section IV.

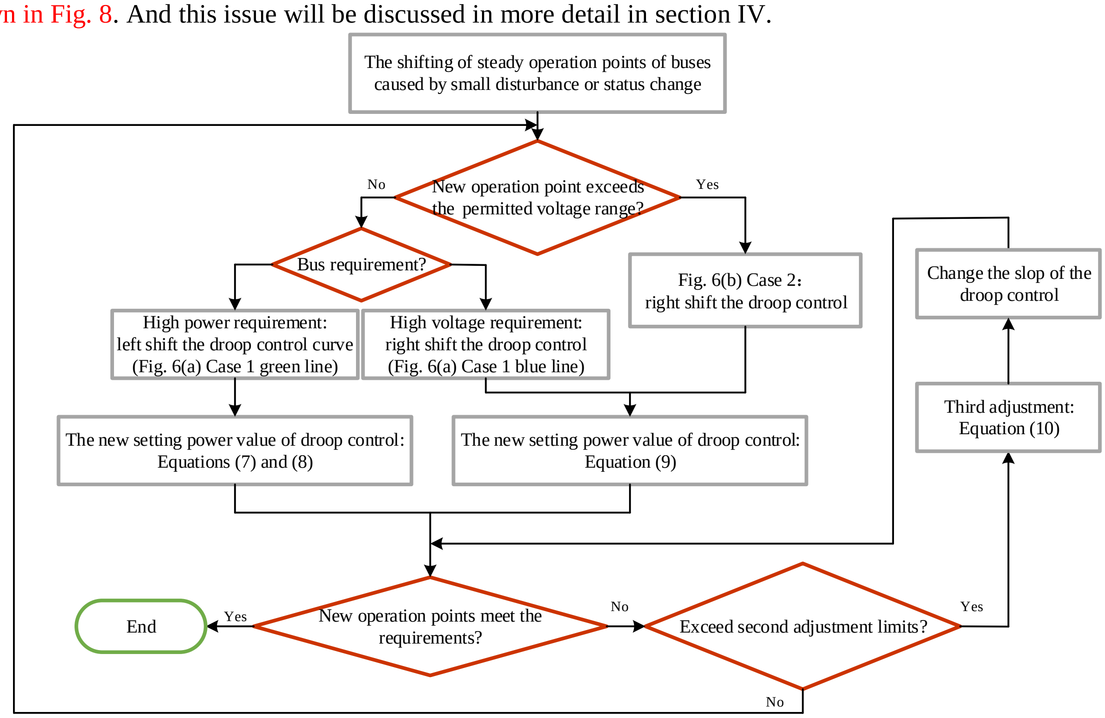


Fig. 8 The control block diagram of automatic adjustment

4. POWER FLOW OF DC DISTRIBUTION NETWORK WITH DROOP CHARACTERISTICS

4.1. *DC power flow controller*

The DC power flow controller can be employed to control the line power flow effectively. The commonly used power flow controller is variable voltage type, variable resistance type and DC/DC converter type. As shown in Fig. 9, this paper takes DC/DC converter type as an example and studies its effect on the power flow of DC distribution network.

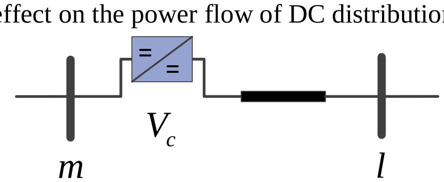


Fig. 9 Diagram of DC power flow controller connecting to distribution network

When the DC power flow controller is connected in series between *m* and *l*, the controller is assumed to be installed close to bus *m*. V'_m is the output voltage of the controller, V_m is the voltage of bus *m* and V_c represents the voltage drop across the controller. So, the relationship between them is as follows:

$$V'_m = V_m - V_c \tag{11}$$

In order to change the power of line *m-l* to meet the demand, we can adjust V_c that is calculated by equations of power flow.

4.2. Power flow equation of DC distribution network with droop characteristic

For a DC distribution network with n terminals, Assume the terminal voltage vector is $V=[V_0, V_1, \ldots, V_{n-1}]$, the terminal current vector is $I=[I_0, I_1, \ldots, I_{n-1}]$, the No. 0 bus is the slack bus, and the conductivity matrix is G. Then $I=GV$, where:

$$G = \begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1n} \\ G_{21} & G_{22} & \cdots & G_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ G_{n1} & G_{n2} & \cdots & G_{nn} \end{bmatrix} \tag{12}$$

The diagonal element G_{ij} is the self-conductance and the non-diagonal element $G_{ij}=G_{ji}$ is mutual conductance.

Assume the terminal power vector is $P=[P_1, P_2, \ldots, P_n]$. Then the power vector can be calculated by voltage vector and current vector, as shown below:

$$P = \begin{bmatrix} V_0 I_0 \\ V_1 I_1 \\ \vdots \\ V_{n-1} I_{n-1} \end{bmatrix} = \begin{bmatrix} V_0 & 0 & \cdots & 0 \\ 0 & V_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & V_{n-1} \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ \vdots \\ I_{n-1} \end{bmatrix} \tag{13}$$

Obviously, the power flow equations of bus *m* and *l* are different from those in (13) and need to be amended when the DC power flow controller is installed. The amended equations of bus *m* and *l* are as follows:

$$\begin{aligned} P_l &= V_l \sum_{j=0, j\neq m}^{n-1} V_j G_{kj} + V_l V'_m G_{lm} = V_l \sum_{j=0}^{n-1} V_j G_{lj} - V_l V_c G_{lm} \\ P_m &= V_m \sum_{j=0, j\neq l}^{n-1} V_j G_{mj} + V'_m V_l G_{ml} = V_m \sum_{j=0}^{n-1} V_j G_{mj} - V_l V_c G_{ml} \end{aligned} \tag{14}$$

The power of the branch *m-l* is P_{ml}:

$$P_{ml} = V_m (V_m - V_c - V_l) G_{ml} \tag{15}$$

Hence, the correct power flow equations are as follows:

$$\begin{cases} P_i = V_i \sum_{j=0}^{n-1} V_j G_{ij} \quad, \; i \neq m, l \\ P_l = V_l \sum_{j=0}^{n-1} V_j G_{lj} - V_l V_c G_{lm} \\ P_m = V_m \sum_{j=0}^{n-1} V_j G_{mj} - V_l V_c G_{ml} \\ P_{ml} = V_m (V_m - V_c - V_l) G_{ml} \end{cases} \tag{16}$$

Besides, the voltage-power relationship is also determined by the droop characteristics:

$$P_i = f_i(V_i) \tag{17}$$

where $f_i(V)$ is the droop characteristic function of bus *i*. For single source or load connection bus, $f_i(V)$ is just simple droop control curve. But for multi-sources and multi-loads bus, $f_i(V)$ has multiple segments.

Combining the grid parameter characteristics in (16) and the droop control characteristics in (17), the operating point of bus can be obtained. In the process of solving, equations of bus power and droop characteristic, the voltage of slack bus, the power of constant power bus and the power of the branch with the power flow controller are known. The power of slack bus, the voltage of constant power bus, the voltage of the power flow controller and the power and voltage of droop control buses.

Assuming k is the number of iterations, the deviation of power $\Delta P'_{n-1}$ meets the following equations.

$$
\begin{cases}
\Delta P_i^{(k)} = P_{i\_ref} - V_i^{(k)} \sum_{j=0}^{n-1} V_j^{(k)} G_{ij} \quad , \ i \neq 0, m, k \\
\Delta P_l^{(k)} = P_{l\_ref} - V_l^{(k)} \sum_{j=0}^{n-1} V_j^{(k)} G_{lj} + V_l^{(k)} V_c^{(k)} G_{lm} \\
\Delta P_m^{(k)} = P_{m\_ref} - V_m^{(k)} \sum_{j=0}^{n-1} V_j^{(k)} G_{mj} + V_l^{(k)} V_c^{(k)} G_{ml} \\
\Delta P_{ml}^{(k)} = P_{ml\_ref} - V_m^{(k)} (V_m^{(k)} - V_c^{(k)} - V_l^{(k)}) G_{ml}
\end{cases} \tag{18}
$$

$$
\Delta P'^{(k)} = \begin{bmatrix} \Delta P_{(n-1)\times 1}^{(k)} \\ \Delta P_{ml}^{(k)} \end{bmatrix} = -J^{(k)} \Delta V'^{(k)} = -J^{(k)} \begin{bmatrix} \Delta V_{(n-1)\times 1}^{(k)} \\ \Delta V_c^{(k)} \end{bmatrix} \tag{19}
$$

$$
J = \begin{bmatrix} H_{(n-1)\times(n-1)} & 0_{(n-1)\times 1} \\ L_{1\times(n-1)} & \dfrac{\partial \Delta P_{ml}}{\partial V_c} \end{bmatrix}_{n\times n} \tag{20}
$$

Where $P_{(n-1)\times 1}$ and $V_{(n-1)\times 1}$ are the power and voltage vector of buses except the slack bus, $V_{n\times 1}$ is the voltage vector containing the voltage of power flow controller, J is a Jacobian matrix. H and L satisfy the following formula (21):

$$
\begin{cases}
H_{ij} = \dfrac{\partial \Delta P_i}{\partial V_j} \\
L_{ij} = \dfrac{\partial \Delta P_{ml}}{\partial V_j}
\end{cases} \tag{21}
$$

The modified voltage:

$$
V^{(k+1)} = V^{(k)} + \Delta V^{(k)} \tag{22}
$$

For the power flow equation shown in (18) and (19), the reference value of the power needs to be updated the droop characteristic function (17) after each iteration until the final calculation results converge. The secondary adjustment of droop characteristic can be performed after each calculation, and the voltage deviation can be mitigated within the effective range. Simultaneously, the additional droop slope adjustment (the third adjustments) can also be considered. So, calculation diagram can be obtained as shown in Fig. 10. The figure is formed by three parts: basic adjustment (1), secondary adjustment (2) and third adjustment (3).

4.3. Influence of droop characteristic on source/ load operation status

Regarding the power flow equation and algorithm, the result of bus voltage and power is associated to the bus droop characteristics. Thus, the actual calculated power result may not be consistent with the dispatching requirement.

Considering a DC distribution network with n terminals, its line transmission power can be calculated with the head end and terminal end voltages. Assume that the head end and terminal end of a transmission line are i and j, the line resistance is r_{ij}, and the head end and terminal end voltages are V_i and V_j. Then the transmission power from i to j is:

$$
P_{ij} = V_i (V_i - V_j) / r_{ij} \tag{23}
$$

It's obvious that the operation points of i and j can be calculated by the previous power flow algorithm. Thus, V_i and V_j are directly related to the bus converter droop characteristics. When the power setting values of i and j are constant, the droop slope variation of bus i and j will definitely influence the transmission power. Conversely, if the power flow cannot meet the power requirement, the droop slope can be utilized to adjust the power flow.

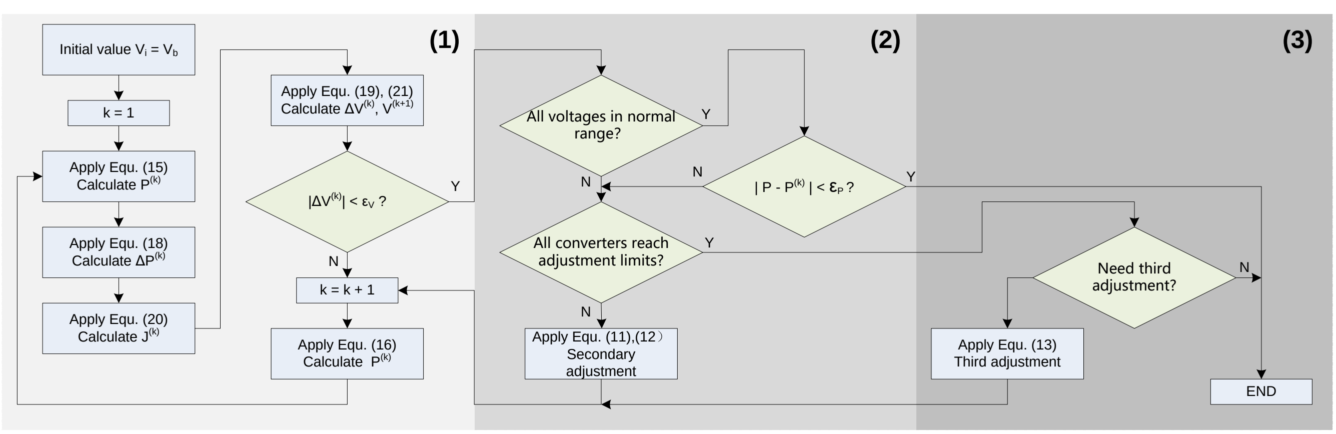


Fig. 10 Power flow calculation diagram of droop control based DC distribution network

For an actual converter, the droop slope is normally solidified in the converter control algorithm. Although it can be set as a tuning parameter, due to the influence of the converter's power electronic switching characteristics, the change of the droop slope parameter will inevitably cause some unpredictable electromagnetic transient processes. Therefore, it is necessary to study the electromagnetic transient process of the system before giving a suitable suggestion.

5. CASE STUDY

The steady-state analysis of DC distribution network in this paper is based on the DC power flow equations considered droop control and power flow controller. And the equations are derived on the premise that a bus connected with the superior grid is referred to as the slack bus. According to the universality of power flow equations, they could be applied to the steady-state analysis of DC distribution networks with different configuration and structure. Here, this paper presents a topological structure of a simplified eight-terminal DC distribution network based on a multi-terminal HVDC performance test transmission network of the International Conference on Large Power Grid (CIGRE) [22]. The topological structure is shown in Fig. 11. The voltage level of the main bus is 10 kV, and all lines are measured in kilometres. “Bb” is the bipolar bus, “Cb” represents the AC/DC converter station and “Cd” is DC/DC converting station. Besides, all the parameters of lines and converting stations are shown in the appendix. Specifically, the droop control of bus has the dead area, the normal range of voltage operation is ±3%, and the range of dead area is ±1.5%. The capacity of converter station “Cb-B1” and “Cd-B1” are 10 MVA and 2MVA.

On the basis of the power flow algorithm in section III, three cases are used for verification:

I. The influence of power flow controller on DC distribution network with droop control.

II. The regulation effect of the secondary adjustment proposed in this paper on the running state of DC distribution network.

III. The regulation effect of the third adjustment proposed in this paper on the running state of DC distribution network.

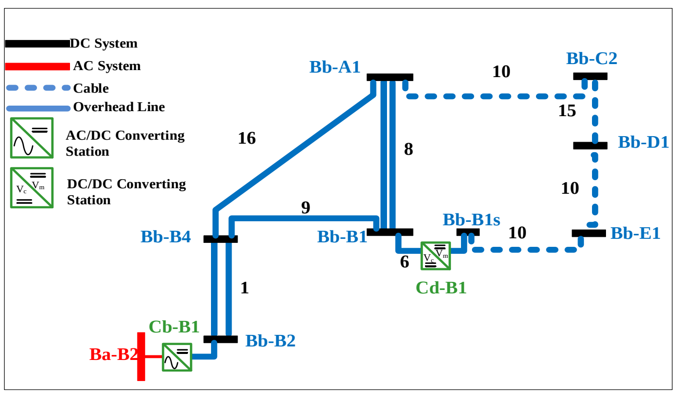


Fig. 11 Topological diagram of 8-terminal flexible DC distribution network

5.1. *Case I*

In case I, the load of DC distribution network is relatively light, and all buses are in a normal operation state. When the control parameters are 0.4 MW and 0.8 MW, the calculation results are shown in Tab.1, where positive power represents power generation, and negative power is power utilization.

When the control parameter of the power flow controller is 0.4 MW, the final powers of bus 2, 3, 4, 5, 6 and 8 are the same as the expected power, which means the bus converters are running in the dead area. While the final power of bus 7 is -2980453 W, which is different from the expected power, resulting from the droop control. Additionally, the transmission power of the line 7-8 is 0.4 MW, achieving the expected goal of power flow controller.

When the control parameter of the power flow controller is 0.8 MW, the voltages of buses except slack bus are adjusted and are still in normal operating range. The final power of bus 5 is -2997663 W, which deviates from the expected power. Obviously, the operation point of bus 5 runs beyond the dead area. Besides, the transmission power of the line 7-8 is 0.8 MW, achieving the expected goal of power flow controller.

Additionally, when the control parameter of DC power flow controller on the line 7_8 changes from 0.4 MW to 0.8 MW, voltage deviations of bus 7 and 8 change from -208.64 V and 21.54 V to -267.02 V and 24.48 V. Obviously, the increase of transmission power of DC power controller results in the increase of voltage difference between bus 7 and bus 8 (230.18 V to 291.5 V). Meanwhile, due to the droop control of converter, the bus voltage deviation is regulated within the permitted range, reflected in the distribution of the voltage difference to both terminals of the line 7_8. And the power of bus 7 cannot meet the need of load because of the bus voltage drop.

Above all, the power flow controller in DC distribution network can effectively control the power flow within the normal operating range. And it is clear that the change of the transmission power of DC power controller causes the change of bus voltage and power of both terminals, which are also affected by droop control.

5.2. Case II

In case II, the load of DC distribution network is relatively heavy, and the expected powers of bus 2 and 7 change to -3 MW and -4 MW, resulting the voltages of part buses go over the limit. The voltage deviation can be relieved by the secondary adjustment of the droop control so that the bus can operate within the normal range. The results of power flow before and after the secondary adjustment are shown in Tab.1.

Before the secondary adjustment of the converter, the actual voltage of bus 7 is 9626.90 V and the deviation of voltage is -373.10 V, which is beyond the normal operating range and affects the operation of the load connected to bus 7. After the secondary adjustment, the voltage of bus 7 becomes 9850V and the deviation of voltage is -150.00 V, which satisfies the normal operating range. Additionally, the voltage deviations of the other buses are adjusted, which are still in normal range. And the transmission power of DC power controller is unchanged, 0.4 MW.

In this case, we can clearly see that the voltage of bus can be regulated back to the normal operation state by the secondary adjustment of droop control when the change of load leads to the abnormal voltage, and the operation of DC power flow is free from the adjustment of droop control.

5.3. Case III

In case III, this paper verifies the regulating ability of the third adjustment with the heavy load condition of case II. As shown in Tab.1, the actual power of buses 5 and 7 are -2974513 W and -3925633 W, meanwhile the power deviations are 25487 W and 74367 W before the third adjustment. The third adjustment needs to be made to reduce the power deviations due to the high power demand of buses 5 and 7. After the third adjustment, the actual power of buses 5 and 7 are -2999568 W and -3999208 W and the power deviations become to 432 W and 792 W. At the moment, the droop slope of bus 5 changes from -0.003 to -0.2, and the droop slope of bus 7 changes from -0.003 to -0.3.

To reduce the power deviations of buses 5 and 7, the droop slopes of converters are changed, leading to the increase of voltage deviation. Hence, the operation range of bus voltage can be enlarged to improve the controllability of bus voltage. On the other hand, the voltage deviations of buses 5 and 7 become from -226.46 V and -373.10 V to -236.39 V and -387.60 V when the droop slopes become steeper, which means the gentler the slope, the smaller the voltage deviation. Considering the fluctuation of new energy output and load, the droop slope of the new energy buses and the load buses should be as flat as possible to meet the normal operation demand.

Tab. 1 Power flow results of cases

Power flow (Power unit W, Voltage unit V)									
Case	Bus	Number	Expected power	Actual power	Actual voltage	Voltage deviation	Branch	Transmission power	Loss power
Case I (power flow controller 0.4 MW)	Bb-B2	1	/	2079396	10000.00	0	1_2	2079396	1297
	Bb-B4	2	-2000000	-2000000	9993.76	-6.24	2_3	1106216	5881
	Bb-A1	3	-6000000	-6000000	9940.63	-59.37	3_4	-1548511	7279
	Bb-D1	5	-3000000	-3000000	9877.23	-122.77	4_5	2444208	26951
	Bb-B1s	7	-3000000	-2980453	9791.36	-208.64	5_6	-582743	1044
	Bb-B1	8	4000000	4000000	10021.54	21.54	6_7	3416212	35758
	Bb-C2	4	4000000	4000000	9987.36	-12.64	7_8	400000	0
	Bb-E1	6	4000000	4000000	9894.93	-105.07	2_8	-1028117	2857
							3_8	-3351152	27275
Case I (power flow controller 0.8 MW)	Bb-B2	1	/	2064162	10000.00	0	1_2	2064162	1278
	Bb-B4	2	-2000000	-2000000	9993.81	-6.19	2_3	1198261	6901
	Bb-A1	3	-6000000	-6000000	9936.26	-63.74	3_4	-1155975	4060
	Bb-D1	5	-3000000	**-2997663**	9842.99	-157.01	4_5	2839965	36505
	Bb-B1s	7	-3000000	-2960994	9732.98	-267.02	5_6	-194203	117
	Bb-B1	8	4000000	4000000	10024.48	24.48	6_7	3805790	44796
	Bb-C2	4	4000000	4000000	9971.16	-28.84	7_8	800000	0
	Bb-E1	6	4000000	4000109	9848.91	-151.09	2_8	-1135377	3485
							3_8	-3652665	32433
Case II (before the 2nd adjustment)	Bb-B2	1	/	4052768	10000.00	0.00	1_2	4052769	4927
	Bb-B4	2	**-3000000**	-3000000	9987.84	-12.16	2_3	1608630	12451
	Bb-A1	3	-6000000	-6000000	9910.53	-89.47	3_4	-585460	1047
	Bb-D1	5	-3000000	-2974513	9773.54	-226.46	4_5	3413493	53194
	Bb-B1s	7	**-4000000**	-3925633	**9626.90**	**-373.10**	5_6	385786	467
	Bb-B1	8	4000000	4000000	10003.00	3.00	6_7	4386201	60569
	Bb-C2	4	4000000	4000000	9928.26	-71.74	7_8	400000	0
	Bb-E1	6	4000000	4000883	9761.70	-238.30	2_8	-560788	851
							3_8	-3818361	35626
Case II (after the 2nd adjustment)	Bb-B2	1	/	4001930	10000.00	0.00	1_2	4001930	4805
	Bb-B4	2	**-3000000**	-3000000	9987.99	-12.01	2_3	1582371	12048
	Bb-A1	3	-6000000	-5998695	9911.95	-88.05	3_4	-634336	1229
	Bb-D1	5	-3000000	-2951233	9778.70	-221.30	4_5	3364435	51646
	Bb-B1s	7	**-4000000**	-3902870	**9850.00**	**-150.00**	5_6	361556	410
	Bb-B1	8	4000000	4000000	10003.81	3.81	6_7	4362720	59849
	Bb-C2	4	4000000	4000000	9931.15	-68.85	7_8	400000	0
	Bb-E1	6	4000000	4001574	9767.61	-232.39	2_8	-585245	927
							3_8	-3794035	35164
Case III	Bb-B2	1	/	4148651	10000.00	0.00	1_2	4148651	5163

(after the 3rd	Bb-B4	2	-3000000	-3000000	9987.55	-12.45	2_3	1658171	13231
adjustment)	Bb-A1	3	-6000000	-6000000	9907.86	-92.14	3_4	-490778	736.092
	Bb-D1	5	-3000000	**-2999568**	9763.61	-236.39	4_5	3508486	56259
	Bb-B1s	7	-4000000	**-3999208**	9612.40	-387.60	5_6	452659	645
	Bb-B1	8	4000000	4000000	10001.47	1.47	6_7	4462044	62836
	Bb-C2	4	4000000	4000000	9922.72	-77.28	7_8	400000	0
	Bb-E1	6	4000000	4010030	9749.70	-250.30	2_8	-514684	717
							3_8	-3864281	36508

6. CONCLUSIONS

In this paper, the operating mechanism of the droop control of the converter in a DC distribution network is discussed, and the steady-state algorithm of the DC distribution network with droop characteristics is given. The conclusions are as follows:

- The actual operation point of the buses in the DC distribution network is determined by the internal characteristics (droop control characteristics) and the external characteristics (network characteristics) of the converter operation. Different types of buses have different requirements for their droop characteristics. Considering the fluctuation of new energy output and load conditions, the droop slopes of the new energy bus and the load bus should be as flat as possible to meet the normal operation demand.
- In the DC distribution network, when a bus accesses multiple converters, the droop control of the bus will be determined by the characteristics of multiple converters as a multi-level segmentation function, which can be used to qualitatively analyze the parallel operation feasibility of multi-converters.
- For the problem of bus voltage deviation and power deviation caused by the power fluctuation in the DC distribution network, the secondary adjustment of the droop characteristic parameter can be used to alleviate the deviation in the adjustment range. For the third adjustment, this paper gives the method of participating in the main adjustment process and conducts a case study.
- Compared with traditional master-slave control method, the droop control method can realize the flexible and stable operation of the DC distribution network under different conditions without centralized communication. Besides, this droop control method also has strong adjustment ability.

APPENDIX

TABLE A
THE PARAMETER OF THE DC DISTRIBUTION NETWORK

Bus	Bus number	Bus droop slope	Branch	Line resistance /Ω
Bb-B2	1	None	1_2	1*3e-2
Bb-B4	2	-0.01	2_3	16*3e-2
Bb-A1	3	-0.01	3_4	10*3e-2
Bb-D1	5	-0.003	4_5	15*3e-2
Bb-B1s	7	-0.003	5_6	10*3e-2
Bb-B1	8	-0.01	6_7	10*3e-2
Bb-C2	4	-0.01	7_8	0
Bb-E1	6	-0.01	8_2	9*3e-2
			8_3	8*3e-2

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